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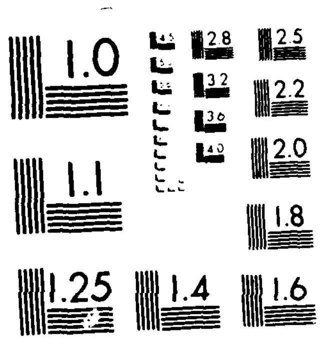

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We study flow fields with concentrations of large vorticity in spots or in slender tube-like regions, known as filaments, by a combination of the method of asymptotic analyses and numerical method.

When the background flow field is potential, the resultant flow is a superposition of the background potential

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flow and that induced by the vorticity distribution in the vortex filament. The matched asymptotic analysis yields the motion of the filament and the decay of the core structure in regions where the core size is much smaller than the other length scales in the flow field. In the regions (or time interval) where the filament merges with itself or with an adjacent filament, numerical solution of the time dependent Navier Stokes equation for the "local" region has to be constructed. The asymptotic analysis provides the initial data and also the appropriate boundary data for the numerical analysis.

When the background flow field is rotational with high Reynolds number, the asymptotic analysis can again be employed to study the decay of high vorticity concentration in spots but the motion of the strong vortical spots is now coupled with the variation of the background vorticity. To study their interaction numerical solutions of the Euler equation are constructed. The particular feature of the numerical scheme is that the grid size and time step depend only on the length scale of the background flow and are independent of the effective size of the spot which can be much smaller than the the grid size.

Vortex Flows

Final Report

by

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Summary

We study flow fields with concentrations of large vorticity in spots or in slender tube-like regions, known as filaments, by a combination of the method of matched asymptotic analyses and numerical method.

When the background flow field is potential, the resultant flow is a superposition of the background potential flow and that induced by the vorticity distribution in the vortex filament. The matched asymptotic analysis is used to study the motion of the filament and the decay of the core structure in regions where the core size is much smaller than the curvature of the filament and the distance to an adjacent filament. In the local regions (or in a time interval) where a part of the filament merges with itself or with that of an adjacent filament, numerical solution of the time dependent Navier Stokes equation for the "local" region has to be constructed. The asymptotic analysis provides the initial data and also the appropriate boundary data for the numerical analysis. Numerical results demonstrate that the trajectories of the filaments differ significantly from those predicted by the matched asymptotic analysis after merging as expected.

When the background flow field is rotational with high Reynolds number, the asymptotic analysis can again be employed to study the decay of high vorticity concentration in spots but the motion of the strong vortical spots is now coupled with the variation of the background vorticity. To study their interaction numerical solutions of the Euler equation are constructed. The particular feature of the numerical scheme is that the grid size and time step depend only on the length scale of the background flow and its velocity and the total strength of the vortical spots. They are independent of the effective size of the spot which can be much smaller

than the grid size. Numerical results of the trajectories of vortical spots in a nonuniform shear flow are obtained. The trajectories differ substantially from the corresponding ones in a linear shear flow or in a uniform flow. The differences are due to the interaction of the vortical spot with the variation of the vorticity in the background flow.

Brief Description of the Investigations

In many flows of interest, where vorticity effects are important, the flow field may be considered as inviscid and irrotational everywhere except in slender tube-like regions, called vortex filaments; here the vorticity is concentrated. In the usual classical idealization, where the diameter of the filament is small compared to the other scales characterizing the problem, the crossectional area of the tube is neglected and the slender tube-like filament is reduced to a curved line, called a vortex line, which is submerged in the fluid field.

Classical inviscid theory describing the motion of vortex lines [1] suffers from two essential defects: The violation of the boundedness on the fluid velocity defined on the vortex line and the nondeterminacy of the velocity of the line itself.

In real fluids the velocity field must remain finite everywhere; this means that when, in local regions (the inner regions), the velocity becomes large the gradient of the velocity must also become large. Consequently, viscous terms, no longer negligible in these regions, become the mechanism by which both the velocity gradients are attenuated as well as causing the vorticity to decay. The defects of the classical inviscid theory for vortex motion can therefore be eliminated if the inviscid solution is identified as the leading term of a matched asymptotic solution of the Navier Stokes equations in the region (the outer region) sufficiently

away from the filament. The condition that the velocity should be finite everywhere should enable us to define the velocity of the vortex filament with a decaying vorticity distribution. With this basic premise, the matched asymptotic solutions for a viscous vortex filament submerged in an outer potential flow field were constructed in a series of papers [2-6] from the simple two dimensional problem to the three dimensional problem with large circumferential and axial velocity components in the vortex filament. The general procedures and the essential conclusions of the analyses [2-6] are described in section II of an invited lecture [7]. Formulas for the velocity of the vortex filament and the circumferential and axial velocity variations in the vortex filament are presented. The influences of the vortex filament on the background potential flow are of higher order. Comparisons are made between the matched asymptotic solutions and the relevant patched solutions [8,9,10]. In addition, several applications of the asymptotic solutions to simulated flow fields [11] and jet noise [12,13] are mentioned. Similarity solutions for the core structure are obtained and identified as the asymptotic solutions for long time. The "optimum" similarity solution corresponding to a given initial core structure is defined and their physical meaning is identified.

In section III of [7] investigations for the motion of a vortex filament in a background rotational flow field are outlined. A method of analysis is outlined for the two dimensional problem. Since the governing equations for the background flow are nonlinear and involve the velocity induced by the vortex, the motion of the vortex with a viscous core is now coupled with the temporal variation of the background flow.

In section IV of [7] several numerical solutions of Navier-Stokes equations for merging of vortex rings are

reported. The asymptotic analysis provides the initial data prior to merging. These results, together with previous results for two dimensional problems [14], are employed to establish a practical upper bound for the expansion parameter within which the match asymptotic solutions are applicable.

Based on the matched asymptotic analyses [7], the equations governing the motion of the vortex filament and the decay of its core structure are a system of integral differential equations in two independent variables, the time t and the parameter s which characterizes the center line of the vortex filament. Stability analyses are made for a simplified system of equations which retain the dominant terms in the original set of equations. It is shown that the explicit scheme is always unstable although it is conditionally stable for simple heat flow equations [15]. A modified Du Fort-Frankel (D-F) scheme is shown to be stable. These results were reported as a contributed paper in the SIAM 30th Anniversary meeting July 1982, Stanford, Ca. The title and the abstract are:

<u>Numerical Study of the Motion of a</u>
<u>Viscous Vortex Filament</u>

by John Tavantzis and Lu Ting

Based on asymptotic analysis, the motion of a viscous vortex filament submerged in a background potential flow was shown by A. J. Callegari and L. Ting [SIAM J. Appl. Math., 35 (1978), pp. 148-175) to be governed by a system of integral differential equations in two independent variables, time t and arc length s. For the stability of the finite difference scheme, a simplified model equation of the form $x_t = Bx_{ss}$ is analyzed where x is the position in R^3 and B is a skew-symmetric matrix. It is found, for this system, that the standard explicit scheme is unstable. The modified Du Fort-Frankel scheme is stable and is then adapted to the original system of equations. According to the asymptotic analysis, these equations contain

two parameters which are defined explicitly by the initial data of large circumferential and axial velocity profiles in the inner viscous core. Numerical examples will be presented.

The D-F scheme was further modified to involve five points in the spatial variable instead of the usual three points. Thus for the same degree of accuracy we can use larger spatial grid size. Additional attempts are being made to improve the accuracy by using the modified D-F scheme as the predictor for an iteration scheme. The schemes are being adopted for the original system of integral differential equations. A paper [11] is being prepared to report the numerical scheme, the stability analyses and the numerical examples, demonstrating the effects of the initial shape of the center line of the filament and its initial core structure, i.e., the axial and circumferential velocity components. The paper should be completed in the coming summer and will be sent to the contractor upon its completion.

It was pointed out before [see also Ref. 7] that there are problems for which the asymptotic method is not applicable and numerical solution of the Navier Stokes equation is needed. Since numerical solutions can be constructed only for a bounded domain we have to impose appropriate boundary conditions and assess the error introduced by these conditions as compared to the real conditions corresponding to the flow field in the unbounded domain. Therefore, we have to develop far field behavior of flow field. For the same degree of accuracy of the difference scheme, a more accurate knowledge of the far field behavior will enable us to use a smaller domain thereby to reduce substantially the total number of grid points and the computational time. By using the integral invariants of Truesdell, Moreau and Howard [17,18] the far field behavior is established and reported in [19]. The

summary of the paper is:

> "Unsteady three-dimensional incompressible viscous flow fields induced by initial vorticity distributions are studied. Relevant invariants and decay laws of the moments of vorticity distributions are presented and shown to be useful in the numerical calculation of flow fields in two ways. First, the moments determine the leading terms of the far-field velocity, which can be employed as boundary conditions for the numerical calculation. Secondly, the deviations of the numerical results from the invariants and the decay laws can be used to measure the error of the numerical solution."

The leading three terms in the far field behavior developed in [19] was employed by Weston and Liu [20] to study the roll-up of vortex wakes. They show that their numerical scheme is much more efficient than the scheme [21] imposing stream function = 0 on the boundary.

The far field behavior [19] was also employed in [22] to study the self-merging of a circular vortex ring, i.e., the core size is comparable to the ring radius. The scheme is much more efficient than the previous one used in [7] which used the Poisson integral to evaluate the boundary data. For the study of the merging of two vortex rings while their core sizes are still much smaller than their radii, the size of the domain for the numerical solution can be larger than the merge region but smaller or much smaller than the radii of the rings. Appropriate far field behaviors were developed and then used to specify the boundary data for the numerical solution. Numerical examples for the head-on collision of two rings show that the trajectory of the ring (the point of maximum vorticity) differ from that of the classical inviscid theory and that of the matched asymptotic analysis when the merging of the two rings begins. The points of maximum (or minimum) vorticity begin to move away from each other and the value begins to decrease. Both facts can be attributed to the merging of the vortical cores, i.e., the

mutual cancellation of vorticity in the overlapping region. Similar qualitative experimental observations were reported by Prof. K. Oshima, Tokyo, Japan. Details of those investigations were reported in a paper by Liu and Ting [22]. The abstract of the paper is:

> "Incompressible viscous flow fields induced by initial vorticity distributions with bounded support or exponential decay in the far field are investigated. A numerical scheme for the solution of the vorticity distribution and the velocity field is presented with special emphasis on the treatment of the boundary data. The efficiency of the scheme is demonstrated. The present method has been applied to the study of the merging and collision of vortex rings."

When the background flow is rotational with high Reynolds number, the viscous terms are important only in the region where there are large velocity gradients, i.e., high vorticity concentrations. For two dimensional problems we call them vortical spots. The presence of the vortical spots and their movements will induce variations of the vorticity distribution in the background flow. The variations will in turn change the background flow and the motion of the spots. This interaction between the vortical spots and the background flow will of course be absent if the background flow is of constant vorticity or a potential flow.

Study of the flow field of vortical spots submerged in a rotation flow by numerical solution of the Navier-Stokes equations is very inefficient since the viscous terms are important only in small spots of high vorticity concentration and the grid size would have to be smaller than the size of the spots. Multiple scale analysis is introduced to isolate the viscous decay of vorticity in the spots as solution of the "small" scale variables while the governing equation for the "normal" scale is the inviscid equation, the Euler equation. The decay of the vortical core in each spot can be described by the matched asymptotic solution [7]

while the motion of the vortical spots is coupled with the variation of the vorticity distribution in the background flow and numerical solution of the unsteady Euler equation is required.

In the Euler equation, the spatial variables are of "normal" length scale. The velocity induced by each vortical spot is replaced by the spatial average $\langle \underline{V} \rangle$. In the theory of multiple scale analysis, $\langle \underline{V} \rangle$ is a function of the spatial variables of normal length scale and the average is evaluated over a domain of size much smaller than the normal scale but much larger than the size of a vortical spot. Consequently the grid size for the difference equation should be smaller than the normal length but the grid size and the time step should be independent of the core size.

The value of the average $\langle \underline{V} \rangle$ at a grid point far away from the vortical spot shall agree with that given by the classical inviscid theory for a point vortex. When the distance between a grid point and a vortical spot is of the order of the core size, the average value differs from the classical theory which becomes infinity as their distance vanishes. The question is how to evaluate $\langle \underline{V} \rangle$ at a grid point close to a vortical spot.

One method to evaluate $\langle \underline{V} \rangle$ is proposed in [7]. The classical inviscid theory shall be used to define $\langle \underline{V} \rangle$ for grid points away from vortical spots, i.e., the distance from the grid point to a nearby vortical spot is greater or equal to a half grid size. For those points the maximum $\langle \underline{V} \rangle$ shall be inversely proportional to the half grid size. For a grid point close to a vortical spot, i.e., their distance is less than a half grid size, we evaluate the value of $\langle \underline{V} \rangle$ by an interpolation formula from the values at the neighboring grid points which are known and the value at the center of the nearby vortical spot where $\langle \underline{V} \rangle = 0$.

Based on this method, a numerical code was developed and the results were presented in the 35th annual meeting of APSDFD in November 1982 by Ting and Liu. The title and the abstract of the talk are:

> "Strong Vortical Spots in a Shear Flow. The motion and interaction of strong vortical spots submerged in a background rotational flow are investigated. The reference length of the background flow is much larger than the size of a vortical spot but the velocity near a spot is much larger than the background velocity. The decay of the inner structure of each vortical spot is obtained by the matched asymptotic analysis. The motion of the spots is coupled with the background flow which is governed by the unsteady vorticity transport equation and the Poisson equation for the stream function. Since the vorticity distribution does not decay exponentially in the far field, invarients for the moments of the redistribution of the initial vorticity are derived. The far field behaviors are then generated and employed to specify boundary conditions for the numerical solutions. Numerical schemes are formulated so that the time step is controlled by the reference velocity of the background flow instead of the maximum velocity near a vortical spot. The numerical schemes will be employed to study the motion of vortical spots in the ground shear flow."

Upon careful examination of the numerical examples we noted a small oscillation of the trajectory of vortical spot whenever it crosses over the threshold of a circle around a grid point with radius of a half grid size. This phenomenon is traced to the switching back and forth of the formula for the evaluation of $\langle \underline{V} \rangle$ from the classical inviscid theory to the interpolation formula. In order to provide a smooth transition across the threshold, we evaluate the average $\langle \underline{V} \rangle$ over a square domain. Its size h is of the order of the grid size, say a half or a quarter of a grid size. Therefore h is much less than the normal length scale but is independent of the small core size. Whenever the distance d from the grid point to a nearby vortical spot is larger than h the difference between the average

and the classical theory becomes very small for small h/d, e.g., when $h/d < 0.4$ the difference is less than 1%. Therefore the transition from the classical solution to the average can be made smaller than the error of the difference scheme. A numerical code based on this procedure has been completed and trajectories for the vortical spots differ insignificantly from those obtained from the previous computational code minus the small oscillations in the latter.

Numerical results of the trajectories of the vortical spots submerged in a nonuniform shear flow differ substantially from the corresponding ones in a linear shear flow or in a uniform flow. For the latter two cases, there is no variation of the background vorticity distribution. The differences can therefore be attributed to the interaction of the vortical spots with the vorticity distribution in the background.

Details of these investigations will soon be reported in a paper by Ting and Liu [23]. Copy will be sent to the contractor after completion.

Under the support of a previous ONR contract, the planing of a flat plate at high Froude number was investigated by Ting and Keller [24]. The asymptotic solution of this two dimensional problem was constructed and unique solution was obtained when the effect of the impact on the free surface of the jet thrown up by the plate was included. The effect of the impact was shown to be equivalent to a suitable pressure distribution moving along the free surface by physical arguments.

Recently, these results were reconfirmed by a systematic asymptotic analysis and solutions for the thin wake region trailing the impact region were obtained. In addition, the analysis was carried out for the three dimensional problem. These results were reported in a paper entitled "Surface

waves induced by an impinging jet," by Miksis and Ting [25]. The paper is accepted for publication in the Physics of Fluids. The abstract of the paper is:

> "The oblique impact of a thin or slender jet on the free surface of a semi-infinite stream is studied. The Froude number based on the thickness or radius of the jet is assumed to be large. Based on the matched asymptotic analysis, the leading term of the solution behind the impact area is constructed. The perturbation solution for the stream away from the impact region and its wake is shown to be equivalent by the linear theory of surface waves with a concentrated load at the impact point and a distributed line load along the wake. These equivalent loads are consistent with considerations of mass and momentum balances and other judicial arguments."

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List of Publications

(1) Ting, L., "Studies on the Motion and Decay of a Filament." Lecture Notes in Physics 148, "Advances in Fluid Mechanics," pp. 67-105, Springer-Verlag, 1981.

(2) Liu, C.H. and Ting, L., "Numerical Solution of Viscous Flow in Unbounded Fluid," Lecture Notes in Physics 170, "8th International Conference on Numerical Methods in Fluid Dynamics," pp. 357-363, Springer-Verlag, 1982.

(3) Ting, L., "On the Application of the Integral Invariants and Decay Laws of Vorticity Distributions," J. Fluid Mechanics 127, Feb. 1983, 497-506.

(4) Miksis, M.J. and Ting, L., "Surface Waves Induced by an Impinging Jet." To be published in Physics of Fluids.

Papers in Preparation

(5) Ting, L. and Liu, C.H., "Strong Vortical Spots in a Shear Flow."

(6) Tavantzis, J. and Ting, L., "Numerical Studies of the Motion of Viscous Vortex Filaments."

List of Presentations

(1) Ting, L., "Studies on the Motion and Decay of a Vortex Filament" (Invited Lecture), presented at the Conference on Fluid Mechanics, March 26-28, 1980, Aerodynamics Institute, Aachen, Germany.

(2) Ting, L., "Integral Invariants and Decay Laws of Vorticity Distributions," presented at the 34th meeting of the American Physical Society, Division of Fluid Dynamics, Monterey, CA, Nov. 22-24, 1981.

(3) Liu, C.H. and Ting, L., "Numerical Solution of Viscous Flow in Unbounded Fluid," presented at the 8th International Conference on Numerical Methods in Fluid Dynamics, Rheinisch-Westfälische Technische Hochschule, Aachen, Germany, June 28-July 2, 1982.

(4) Tavantzis, J. and Ting, L., "Numerical Study of a Viscous Vortex Filament," presented in the SIAM 30th Anniversary Meeting, July, 1982, Stanford, CA.

(5) Ting, L. and Liu, C.H., "Strong Vortical Spots in a Shear Flow," presented at the 35th meeting of the American Physical Society, Division of Fluid Dynamics, Nov. 21-23, 1982.

(6) Miksis, M.J. and Ting, L., Surface Waves Induced by an Impinging Jet," presented at the 35th meeting of APS DFD, Nov. 21-23, 1982.